TRINITY COLLEGE

Year 11 Mathematics Specialist
Test 6 2019

Calculator Free

Proof by Induction and Complex Numbers

STUDENT'S NAME ____

DATE: Wednesday 25th September **TIME:** 50 minutes **MARKS**: 46

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

---

1. (3 marks)

State the following recurring decimal as a fraction. It is not necessary to simplify the fraction.

1.28333333333…

2. (2 marks)

For the complex number $z = 3i - 2$, state:

(a) $\text{Re}(z)$ [1]

(b) $\overline{z}$ [1]

3. (3 marks)

Determine the complex solutions to the equation $2x^2 - 4x + 7 = 0$ in their most simplified form.

4. (4 marks)

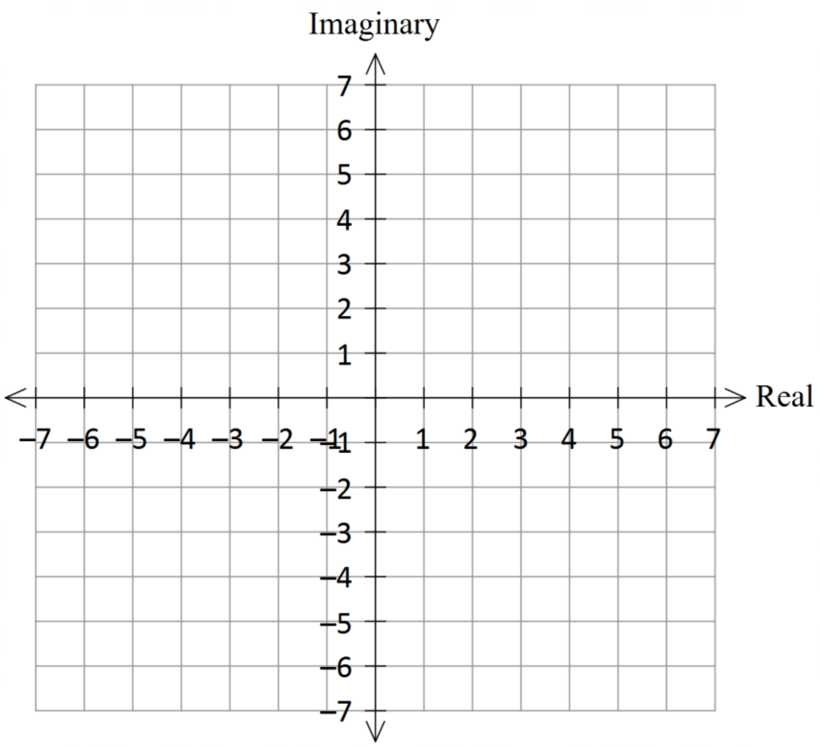
Plot the following complex numbers on the argand diagram below. Label all the points clearly.

(a) $z_1 = 2 + 4i$

(b) $z_2 = -3 + 2i$

(c) $z_3 = \overline{z_1}$

(d) $z_4 = iz_2$



5. (4 marks)

If 3 - 2i is a root of the quadratic equation $x^2 + bx + c = 0$, determine the values of b and c.

6. (7 marks)

If $z = 2 - 5i$ and $w = -3 + 2i$, determine:

(a) $z - 2w$ [2]

(b) $\frac{w}{z}$ [3]

(c) $w\overline{w}$ [2]

7. (4 marks)

Determine a and b if $\frac{(1-3i)^2}{2-i} = a + bi$.

8. (6 marks)

Prove, by mathematical induction, that $4^n - 1$ is divisible by 3 for any positive integer n.

9. (6 marks)

Use mathematical induction to prove the following conjecture:

$1+x+x^2+x^3+...+x^{n-1}=\dfrac{1-x^n}{1-x}$, $n \geq 1$, n a counting number.

10. (6 marks)

Use mathematical induction to prove the following conjecture:

$$\frac{1}{1(3)}+\frac{1}{2(4)}+\frac{1}{3(5)}+\ldots+\frac{1}{n(n+2)}=\frac{3}{4}-\frac{2n+3}{2(n+1)(n+2)}, \quad n \geq 1, n \text{ a counting number.}$$